Holy Spirit University of Kaslik
Faculty of Arts and Sciences

MAT 418

Worksheet 3 - Numerical differentiation and Integration

1. Consider the data set

i	0	1	2	3	4	5	6	7	8
x_i	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
y_i	1.000	2.122	3.233	4.455	5.566	-1.000	-1.255	-1.800	-2.000

   (a) Find $f'(0.25)$ by using successively the two-point forward-difference, the two-point backward-difference and the three-point centered-difference formulas for $h = 0.25$

   (b) Find an approximation of $f'(1)$ by using the three-point centered-difference formula for $h = 0.25, h = 0.50$ and $h = 1.00$.
   Ameliorate the error of differentiation by using the first and second order centered Richardson extrapolation operators for $h = 0.25, 0.5, 1$.
   Give the results in a table and the formulas of the used operators.

   (c) Approximate $f'(0)$ and $f'(2)$ for $h = 0.25$

   (d) Find an approximation of $f''(1)$ by using successively the two-point forward-difference, the two-point backward-difference and the three-point centered-difference formulas for $h = 0.25, 0.5$.
   Which formula do we use to approximate $f''(1)$ for $h = 1$?
   Ameliorate the error of differentiation by using the first and second order centered Richardson extrapolation operators for $h = 0.25$
   Give the results in a table and the formulas of the used operators.

2. Consider the data set

i	0	1	2	3	4	5	6	7	8
x_i	0.000	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000
y_i	1.00000	1.110822	1.197923	1.26638	1.319617	1.360059	1.389507	1.409356	1.420735

   (a) Use the three-point centered-difference formula to approximate $f'(0.5)$, then ameliorate the error of differentiation by using the first and second order centered Richardson extrapolation operators (give the used formulas). Complete the following table

h	$\psi_h(\cdot)$	$\psi_h^{(1)}(\cdot)$	$\psi_h^{(2)}(\cdot)$
0.5	$\times$		
0.25	$\times$	$\times$	
0.125	$\times$	$\times$	$\times$

   (b) Calculate $f''(0.5)$ by using the three-point centered-difference formula. Ameliorate the error of differentiation by using the first and second order centered Richardson extrapolation operators (give the formulas of the used operators). Complete the following table

h	$\psi_h(\cdot)$	$\psi_h^{(1)}(\cdot)$	$\psi_h^{(2)}(\cdot)$
0.5	$\times$		
0.25	$\times$	$\times$	
0.125	$\times$	$\times$	$\times$

(c) Find an approximation of $f'(0)$ by using successively the two-point forward-difference formula if $h = 0.25$ and ameliorate the error of differentiation by using the first and second order forward Richardson extrapolation operators. Give the results in the following table and the formulas of the used operators.

h	$\phi_h(\cdot)$	$\phi_h^{(1)}(\cdot)$	$\phi_h^{(2)}(\cdot)$
0.5	$\times$		
0.25	$\times$	$\times$	
0.125	$\times$	$\times$	$\times$

3. Carry out four panels approximations of $I = \displaystyle\int_1^2 \ln\, x dx$ using the Composite Trapezoid Rule and Composite Simpson's Rule.

4. Find the number n of necessary subdivisions of the integration interval $[-\pi, \pi]$,to evaluate within an error of 0.5×10^{-3}, using the Simpson rule, the integral $\displaystyle\int_{-\pi}^{\pi} \cos x dx$.

5. Find the number of panels m necessary for the Composite Simpson's Rule to approximate $I = \displaystyle\int_0^{\pi} \sin^2 x dx$ with 6 correct decimals places.

6. We consider the function f known only for some values of x :

x	0.00	0.25	0.50	0.75	1.00
$f(x)$	0.3989	0.3867	0.3521	0.3011	0.2420

Evaluate the integral $I = \displaystyle\int_0^1 f(x)dx$ using Romberg of the highest possible order. Fill a table of the form

h	$\mathcal{R}(\cdot)$	$\mathcal{R}^1(\cdot)$	$\mathcal{R}^2(\cdot)$
1		$\times$	$\times$
0.5			$\times$
0.25			

7. Let $f(x) = 2^x$.
   (a) Approach $\displaystyle\int_0^4 f(x)dx$ by the Composite Trapezoid Rule using the partition $0, 2, 4$.
   (b) Find another approximation of $\displaystyle\int_0^4 f(x)dx$ using the partition $0, 1, 2, 3, 4$.
   (c) Apply Romberg Integration to obtain a better approximation.