Holy Spirit University of Kaslik
Faculty of Arts and Sciences

MAT 418

Worksheet 2 - Interpolation

1. Use Lagrange interpolation to find a polynomial of degree 3 or less whose graph passes through the following data points and deduce an estimate of the value of $f(1)$.

x_i	-2	0	3	4
$f(x_i)$	5	1	55	209

2. Deduce by a linear interpolation the value of $\sin(0.55)$ and determine a bound on the error of this estimate. Compare this limit with the actual difference between the estimated value and the exact value. Knowing that :

$$\sin(1/2) = 0.4794255 \qquad \text{and } \sin(0.6) = 0.5646424.$$

3. Use divided differences to find the interpolating polynomial which gives the best approximation for the function $f(x) = \ln x$ that passes through the following data points $(x_i, f(x_i))$ and then determine the value of $f(1.2)$

x_i	1	1.5	2	3	3.5	4
$f(x_i)$	0	0.17609	0.30103	0.47712	0.54407	0.60206

4. Prove that the following function is a cubic spline :

$$s(x) = \begin{cases} -x^3 + \frac{17}{2}x^2 - 9x + \frac{3}{2} & -1 < x < 1 \\ 2x^3 - \frac{1}{2}x^2 - \frac{3}{2} & 1 < x < 2 \\ x^3 + \frac{11}{2}x^2 - 12x + \frac{13}{2} & 2 < x < 4 \end{cases}$$

5. Using cubic spline functions, find an approximation of $f(-3), f(1)$ and $f(5)$ by considering

x_i	-6	-4	-2	-1	0	6	7
$f(x_i)$	-121	-25	-1	-1	-1	335	503

6. Find the degree 3 Lagrange polynomial that passes through the following points, then the Newton interpolating polynomial $(x_0, y_0) = (0, 1), (x_1, y_1) = (1, 3), (x_2, y_2) = (2, 4)$ and $(x_3, y_3) = (3, 0)$.
7. By using the divided differences method, find a polynomial p of degree 3 or less such that :

$$p(1) = 0, \;\; p(2) = 7, \;\; p(-2) = -9, \;\; p(0) = -1$$

8. We aim to interpolate a function $f \in C^4([a, b])$ by a polynomial p_3 of degree ≤ 3 satisfying :

$$f(a) = p_3(a), \qquad f'(a) = p_3'(a), \qquad f(b) = p_3(b), \qquad f'(b) = p_3'(b).$$

Prove the existence and the unicity of p_3 by calculating it under the form :

$$p_3(x) = \alpha_0 + \alpha_1(x - a) + \alpha_2(x - a)^2 + \alpha_3(x - a)^2(x - b).$$

9. Consider the function $f(x) = \cos(x)$ and the interpolation nodes $x_i = 0, 1, 3, 3.5, 5$. Calculate $f(3.14159)$ by a cubic spline interpolation with free extremities.

10. Consider the interpolation polynomial $P_2(x)$ of $f(x) = x^3$ at

$$(0, 0), (-1, -1), (1, 1).$$

Without determining the equation of $P_2(x)$, calculate $E(x) = f(x) - P_2(x)$, and determine in which points of $[0, 1]$, $E(x)$ is the maximum.

11. Find the line that best fits the 3 data points $\{(t_i, y_i) = (1, 2), (-1, 1), (1, 3)\}$.

12. Solve the least squares problem

$$\begin{pmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -3 \\ 15 \\ 9 \end{pmatrix}$$

13. (a) Fit the recorded temperatures in Washington, D.C. on January 1,2001, as listed in the following table, to a periodic model

time of the day	12mid	3am	6am	9am	12noon	3pm	6pm	9pm
t	0	$1/8$	$1/4$	$3/8$	$1/2$	$5/8$	$3/4$	$7/8$
temp(C)	-2.2	-2.8	-6.1	-3.9	0.0	1.1	-0.6	-1.1

(b) Fit the temperature data to the improved model.

14. Use model linearization to find the best least squares exponential fit $y = C_1 e^{C_2 t}$ to the following world automobile supply data

year	1950	1955	1960	1965	1970	1975	1980
cars$\times 10^6$	53.05	73.04	98.31	139.78	193.48	260.20	320.39

15. Fit the drug concentration model with the measured level of the drug norfluoxetine in a patient's bloodstream, given the following table

hour	1	2	3	4	5	6	7	8
concentration(ng/ml)	8	12.3	15.5	16.8	17.1	15.8	15.2	14.0